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COMPARISON OF THE EXTREME RESPONSES FROM DIFFERENT METHODS OF SIMULATING WAVE KINEMATICS

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Abstract. Linear random wave theory (LRWT) is frequently used to simulate water particle kinematics at different nodes of an offshore structure from a reference surface elevation record. However, it is well known that LRWT leads to water particle kinematics with exaggerated high-frequency components in the vicinity of mean water level (MWL). Methods have been introduced to overcome this problem of high kinematics above the MWL consists of using linear wave theory (such as Wheeler, vertical stretching, effective node elevation and effective water depth methods) can be used to provide a more realistic representation of near-surface wave kinematics. There is promising as there is some evidence that the water particle kinematics from the Wheeler method are underestimated and that those from the vertical stretching method are somewhat exaggerated. In this paper, the comparisons of the probability distributions of extreme values from different methods of simulation wave kinematics are investigated by using Monte Carlo simulation procedure.

1 INTRODUCTION

For an offshore structure, wind, wave and gravitational forces are all important sources of loading. The dominant load, however, is normally due to wind-generated random waves. It is therefore of great importance to calculate the wave loads on the structure accurately. Morison's equation [1] is frequently used to calculate wave loads on the cylindrical members of an offshore structure from the wave-induced water particle kinematics. It can therefore be concluded that the accurate estimation of wave-induced water particle kinematics is a key step for accurate prediction of wave loads on the structure.

Linear random wave theory (LRWT) is frequently used to calculate wave-induced water particle kinematics at different nodes of an offshore structure from a simulated surface elevation record by using appropriate transfer functions. It is, however, well known that linear wave theory gives unacceptable results near the free surface, especially for high- frequency wave components. To overcome this deficiency, a common industry practice for evaluation of wave kinematics in the free surface zone consists of using linear wave theory in conjunction with empirical techniques to provide a more realistic representation of near-surface wave kinematics. The empirical techniques popular in the offshore industry include Wheeler stretching [2], linear extrapolation and delta stretching [3] and vertical stretching [4]. Couch and Conte [5] offer a review of these techniques.

More accurate results can be obtained from the Hybrid Wave Model, which is a second order random wave theory [6]. In one study [7], water particle kinematics near the free surface zone from some laboratory experiments were compared with predictions from different methods. It was concluded that the Hybrid Wave Model was more accurate than either the Wheeler method or the linear extrapolation technique. The results indicated that while the linear extrapolation method overestimated the water particle kinematics, the reverse was true for the Wheeler method. Longridge et al [8] made similar conclusions from analysis of laboratory data. They also concluded that both the Wheeler and the linear extrapolation methods are sensitive to the cut-off frequency of the surface elevation frequency spectrum. In other words, they lead to exaggerated water particle kinematics for high-frequency wave components. Donelan et al [9] also concluded from analysis of laboratory data that both direct application of LRWT and the linear extrapolation method greatly overestimate water particle velocities in the near surface zone and that they are both sensitive to the choice of cut-off frequency.

Couch and Conte [5] used water particle kinematics from the Hybrid Wave Model together with those from various stretching techniques to compare the predicted response of the Cognac platform with corresponding measured response data. They concluded that the Hybrid Wave Model leads to more accurate response predictions and that the Wheeler method was better than delta and vertical stretching techniques, which overestimate the response particularly at high frequencies. This conclusion is different from other studies, which indicate that the Wheeler method underestimates the water particle kinematics under wave crests. It should, however, be noted that Morison's drag and inertia coefficients used in this study were 0.90 and 2.3, respectively, which are somewhat high, and that the response evaluation did not account for variation of wave kinematics in the horizontal direction. The effect of wave directionality was not considered, either. It is, therefore, reasonable to conclude that the response has been overestimated due to the foregoing reasons and that this has compensated for the underestimation of water particle kinematics by the Wheeler method.

Although more data is required to make reliable conclusions, it is generally believed that the Wheeler stretching technique underestimates the water particle kinematics under wave crests while other stretching methods tend to overestimate it. It is therefore desirable to come up with a method that resolves this problem. While the Hybrid wave model is more accurate, it is computationally very demanding and is mostly suitable for research purposes. Ideally, a modified form of LRWT which could possibly lead to more accurate results compared with other stretching methods is required. To this end, two new methods, the effective node elevation and the effective water depth methods, have been introduced in this study [10,11]. The results indicate that the water particle kinematics predicted from these methods lie between corresponding values from the Wheeler and the vertical stretching methods. Ideally, comparisons with high-quality laboratory and field data, or corresponding results from the more accurate Hybrid wave model, are necessary to determine the level of accuracy of the proposed procedures in comparison with other techniques.

In this paper, the extreme structural responses for three different sea states and zero-current cases are calculated from four different methods of simulating water particle kinematics (vertical stretching, Wheeler stretching, effective node elevation and effective water depth methods) to investigate by how much they differ from each other. It is shown that the Wheeler and the vertical stretching methods, both popular in the industry, lead to significantly different estimates of the 100-year responses. Furthermore, two new methods for predicting water particle kinematics are introduced whose predicted 100-year responses lie between those from the Wheeler and the vertical stretching methods, and hence may be considered to be more appropriate for practical application.

2 TEST STRUCTURE AND RESPONSES

The test structure used in this paper is a fixed platform in a water depth of 110m. The general outline of the platform is shown in Figure 1. The platform is composed of four vertical legs, where the diameter of each leg is 1.5m with a wall thickness of 40mm. As shown in the figure, the distributed hydrodynamic load on each leg is represented by 30 point loads so that the total number of nodal loads on the four legs is 120. The dimensions of the platform deck are 35m*38m. The member surfaces were assumed to be rough and hence the drag and inertia coefficients were taken to be 1.05 and 1.20, respectively. The total mass of the topsides and the four legs (including the added hydrodynamic mass for the four legs of the structure) is 17665 Tonnes.

The foregoing test structures were subjected to various uni-directional sea-states simulated from Pierson–Moskowitz (P–M) frequency spectrum. The waves were assumed to propagate in the global Y direction (Figure 1). In this study, the following definition of the P–M spectrum [12] has been used:

$$G_{\eta\eta}(f) = \frac{{H_s}^2}{4\pi {T_z}^4 f^5}\ exp\left(-\frac{1}{\pi {T_z}^4 f^4}\right) \tag{1}$$

where $G_{\eta\eta}$ is the surface elevation frequency spectrum, H_s is the significant waveheight in meters, T_z is the mean zero-upcrossing period in second and f is the wave frequency in Hz,.

Surface elevation and corresponding water particle kinematics at different structural nodes were simulated according to linear random wave theory (LRWT). All the water particle kinematics have been multiplied by a wave kinematics factor of 0.95 to account for wave directionality in the sea. The mean zero-upcrossing period (in seconds) for each sea state was taken to be $T_z = 3.55\sqrt{H_s}$ with H_s denoting the significant waveheight in meters. Furthermore, each response has been calculated for three different environmental conditions represented by H_s = 15m, 10m, and 5m, respectively. Surface elevation frequency spectra for H_s = 15m, 10m and 5m are shown in Figure 2. The following responses were chosen for investigation: base shear and overturning moment.

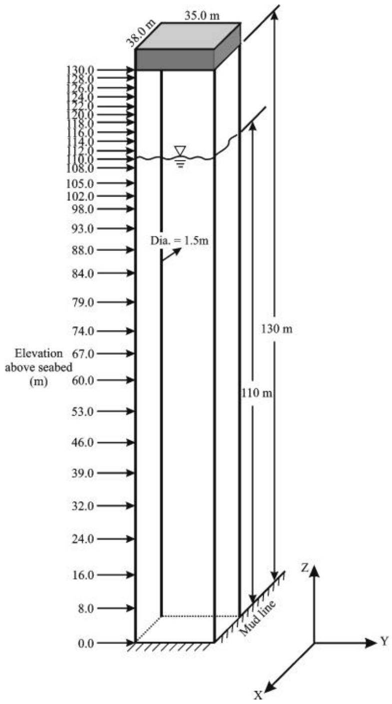

Figure 1: Schematic diagram of the test structures.

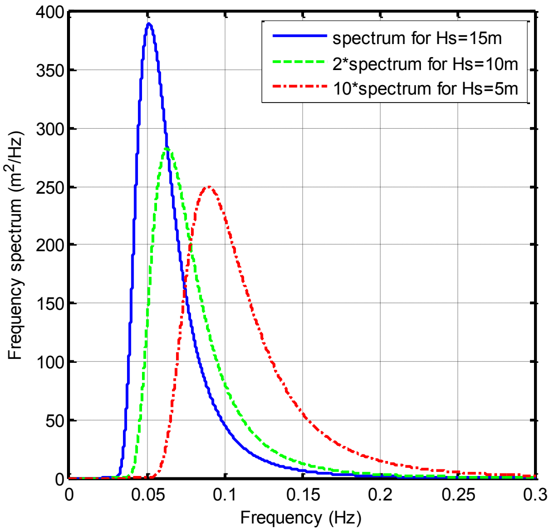

Figure 2: Water surface elevation frequency spectra for three different sea states.

3 WAVE LOADING ON CYLINDRICAL MEMBERS OF OFFSHORE STRUCTURES

According to Morison's equation, the wave-induced horizontal force per unit length on a vertical submerged cylinder (cylinder diameter / wavelength < 1/5) is the sum of a nonlinear drag component and a linear inertial component. That is,

$$F(t) = F_{drag}(t) + F_{inertia}(t) \tag{2}$$

where drag and inertial components of fluid loading are, respectively, defined as

$$F_{drag}(t) = k_d u(t)|u(t)| \tag{3}$$

$$F_{inertia}(t) = k_i \dot{u}(t) \tag{4}$$

$$k_d = \frac{1}{2} C_d \rho D \quad \text{and} \quad k_i = \frac{1}{4} C_m \rho \pi D^2 \tag{5}$$

where C_d and C_m are empirical drag and inertia coefficients; ρ is the fluid density; D is the leg cylinder diameter; and $u(t)$ and $\dot{u}(t)$ are the horizontal component of water particle velocity and acceleration, respectively. Further details about Morison's equation can be found in Sarpkaya and Isaacson [13] and Moe [14]. The assumption made in this paper is that Morison's equation with constant C_d and C_m values can adequately describe the in-line wave forces for a given sea state.

4 EVALUATION OF QUASI-STATIC RESPONSE BY TIME SIMULATION PROCEDURE

In summary, the steps taken to calculate the quasi-static response are as follows:

1. Assume a suitable surface elevation frequency spectrum, such as the Pierson-Moskowitz spectrum defined by its significant wave height, H_s and mean zero-upcrossing period, T_z.
2. Use linear random wave theory (LRWT) to simulate a surface elevation record at an arbitrary reference point from the given frequency spectrum for a given period of time (4.5 hours in this study). According to LRWT, uni-directional seas can be modelled as the sum of a large number of progressive linear waves (wavelets) of different amplitudes travelling in the same direction with random phase angles [15]. Then, the surface elevation at point y at time t can be modelled as:

$$\eta(y,t) = \sum_{i=1}^{NW} A_i \cos(2\pi f_i t - k_i y - \varphi_i) \tag{6}$$

where NW is the total number of wavelets used in the simulation, f_i are a set of equally-spaced discrete wave frequencies and k_i are their associated wavenumbers. Parameter φ_i is a random phase angle distributed uniformly in the range $0 < \varphi_i < 2\pi$, and A_i is determined by one of the following two methods: (1) Deterministic Spectral Amplitude technique (DSA) and (2) Non-Deterministic Spectral Amplitude (NSA) technique. That is,

$$(A_i)_{DSA} = \sqrt{2G_{\eta\eta}(f_i)\Delta f} \tag{7}$$

$$(A_i)_{NSA} = (A_i)_{DSA} * \sqrt{\frac{g_i^2 + h_i^2}{2}} \tag{8}$$

where $G_{\eta\eta}(f)$ is the one-sided surface elevation frequency spectrum and Δf is the frequency interval. Parameters g_i and h_i are two independent and standardised Gaussian random variables. Rice [16] has shown that when *NW* approaches infinity, the two models will be equivalent to each other. The differences between the two techniques for finite values of *NW* have been discussed in Tucker et al [17), Grigoriu [18] and Morooka and Yokoo [19]. The NSA technique is more robust and hence has been used in this study.

3. Calculate wave-induced water particle kinematics at different nodes from the surface elevation record by using appropriate transfer functions from linear wave theory. All empirical wave stretching techniques discussed in previous section were adopted for comparison purposes.
4. Calculate the drag and inertial components of Morison loads at each node accounting for load intermittency in the splash zone.
5. Calculate the drag-induced, $\tilde{R}_d$ and inertia-induced, $\tilde{R}_i$ components of the response (quasi-static base shear and overturning moment in this study). As the structural system is assumed to be linear, the total response would then be equal to the sum of the foregoing two components.

5 DERIVATION OF PROBABILITY DISTRIBUTION OF RESPONSE EXTREME VALUES BY THE MONTE CARLO PROCEDURE

For short-term distribution, use the procedure in Section 4 to simulate a response record from a simulated surface elevation record and determine its extreme value. Then repeat the process many times to generate a large sample of response extreme values. Rank all the simulated extreme values from smallest to largest. Then use the following plotting position equation for the Gumbel distribution [17] to estimate the value of the probability distribution for each of the ranked extreme values.

$$Prob(r_{max} < q_n) = P_{r_{max}}(q_n) \approx \frac{n - 0.44}{N + 0.12}, \qquad n = 1, 2, 3, \dots, N \tag{9}$$

where r_{max} denotes the response extreme value, q_n is the *n*th smallest simulated extreme value, and finally *N* is the total number of simulated extreme values.

6 REVIEW OF EXISTING METHODS OF WAVE KINEMATICS

According to LRWT, uni-directional seas can be modelled as the sum of a large number of progressive linear waves (wavelets) of different amplitudes travelling in the same direction with random phase angles [15]. Then, the surface elevation (η) at point x and time t can be modelled as:

$$\eta(x,t) = \sum_{i=1}^{NW} \eta_i(x,t) \sum_{i=1}^{NW} A_i \, cos(2\pi f_i t - k_i x - \varphi_i) \tag{10}$$

where NW is the total number of wavelets used in the simulation, f_i are a set of discrete wave frequencies (Hz) and k_i are their associated wave numbers. Parameter φ_i is a random phase angle distributed uniformly in the range $0 < \varphi_i < 2\pi$, and A_i is the amplitude of the ith wave component. The horizontal water particle velocity (u) at a point with elevation (z) from mean water level (MWL) (assumed to be positive upwards) would then be equal to

$$u(x,z,t) = \sum_{i=1}^{NW} u_i(x,z,t) \tag{11}$$

$$u_i(x,z,t) = A_i(2\pi f_i)\frac{\cosh[k_i(d+z)]}{\sinh(k_i d)}\cos(2\pi f_i t - k_i x - \varphi_i) \tag{12}$$

where d is the (mean) water depth. For high-frequency components, the wave length would be small and deep water condition would apply; therefore, the above equation can be simplified to

$$u_i(x,z,t) = A_i(2\pi f_i)e^{k_i z}\cos(2\pi f_i t - k_i x - \varphi_i) \tag{13}$$

The value of $e^{k_i z}$ is always smaller than unity for negative z values (points below the MWL); however, it grows very rapidly for high k and z values. This would lead to substantial high- frequency components of water particle kinematics at points above the MWL [20].

6.1 Vertical stretching method

As previously mentioned, various techniques have been developed to avoid this problem. The simplest one is the vertical stretching method [4]. In this method, water particle kinematics at points below MWL are calculated from (standard) LRWT, but water particle kinematics above the MWL are taken to be equal to their corresponding values at MWL. In other words, the following relationship is assumed to be valid.

$$u(x,z,t) = u(x,0,t), \quad z > 0 \tag{14}$$

6.2 Wheeler stretching method

In the Wheeler stretching method [2], the following equation is used to replace the vertical coordinate z with an equivalent node elevation which is always negative. That is,

$$z'(x,t) = d\frac{d+z}{d+\eta(x,t)} - d \tag{15}$$

where η is the instantaneous surface elevation at point x and time t. It should be clear from the above equation that z'changes with time and that it is always negative when the point under consideration is inundated, that is when $\eta > z$. When the surface elevation is below the point, z' would be positive, but then, the water particle kinematics must be set equal to zero as the surface elevation is below the point. Therefore, the problem with rapid growth of water particle kinematics for high- frequency wavelets would not arise in the case of Wheeler approach. However, since z' is a function of time, a transfer function could not be established to convert the surface elevation to water particle kinematics. Therefore, water particle kinematics for each wavelet must be calculated separately, and then, the contributions from all wavelets must be added up to calculate the water particle kinematics due to all wavelets. This is in contrast with the LRWT, where transfer function (and hence the very efficient Fast Fourier Transform (FFT) technique) can be used to calculate water particle kinematics from the reference surface elevation record.

6.3 Effective node elevation method

It is, therefore, desirable to introduce an effective node elevation, z_e [10] which is negative but unlike that of the Wheeler method is of constant value. This was the basis of the effective node elevation method. In this technique, the constant effective node elevation, z_e is taken to be the average value of z' when the node is inundated; in other words, when $\eta \geq z$. According to LRWT, η is a mean-zero Gaussian random variable whose standard deviation (σ_η) is equal to $H_s/4$, where H_s is the significant wave height of the sea state. Therefore, the probability density function of the surface elevation is equal to

$$p(\eta) = \frac{1}{\sqrt{2\pi}\sigma_\eta} e^{\frac{-\eta^2}{2\sigma_\eta^2}} \tag{16}$$

Then, the average value of z' (referred to as the effective node elevation), when $\eta \geq z$, would be equal to

$$z_e = E[z'|\eta \geq z] = \frac{\int_z^\infty z'(\eta)p(\eta)d\eta}{\int_z^\infty p(\eta)d\eta} \tag{17}$$

where

$$z'(\eta) = d\frac{d+z}{d+\eta} - d \tag{18}$$

6.4 Effective water depth method

In the effective water depth method, d_e [11] is taken as the average value of the surface elevation above sea bed when the node is inundated; in other words, when $\eta \geq z$. The effective water depth for a particular node is then equal to,

$$d_e = d + E[\eta|\eta \geq z] = d + \frac{\int_z^\infty \eta p(\eta)d\eta}{\int_z^\infty p(\eta)d\eta} \tag{19}$$

According to Eq. (19), $d_e > d$ and also $d_e > d + z$ (where $d + z$ is simply the node elevation above seabed). Therefore, the node elevation with respect to the effective MWL,$z_e = (d + z) - d_e$, would always be negative. Hence, when these values of d_e and z_e are used in the standard Linear Random Wave Theory, large high-frequency water particle kinematics for points above MWL would be avoided. The horizontal water particle velocity would then be equal to

$$u(x,z,t) = \sum_{i=1}^{NW} u_i(x,z,t) \tag{20}$$

$$u_i(x,z,t) = A_i(2\pi f_i)\frac{\cosh[k_i(d_e + z_e)]}{\sinh(k_i d_e)}\cos(2\pi f_i t - k_i x - \varphi_i)$$

$$= A_i(2\pi f_i)\frac{\cosh[k_i(d+z_e)]}{\sinh(k_i d_e)}\cos(2\pi f_i t - k_i x - \varphi_i) \quad (21)$$

Further details of these techniques can be found in Mohd Zaki et al [21].

7 EFFECT OF DIFFERENT METHODS OF SIMULATING WATER PARTICLE KINEMATICS ON THE PROBABILITY DISTRIBUTION OF THE EXTREME RESPONSES

In this study, the Monte Carlo simulation technique has been used to derive the probability distributions of the extreme responses from the four different methods of simulating water particle kinematics based on 20000 simulated records, each of 128sec duration. 20000 simulated extreme values will ensure that the sampling variability is low and would allow any systematic difference between the two distributions to be observed without any ambiguity [22]. Each signal itself is short to reduce the computational effort for the Wheeler method as in this method the very efficient fast Fourier transforms (FFT) cannot be used for evaluation of water particle kinematics from a simulated surface elevation record. It should be noted that in this study the conclusions for both base shear and overturning moments are similar when comparing the results from different H_s values. Therefore, it would better to show a sample of results for both responses to indicate generality of the conclusions.

As an example, the probability distributions of the extreme responses from the four methods for the quasi-static overturning moment with H_s = 15m are compared in Figure 3. A similar comparison for the case of quasi-static base shear with H_s = 5m is given in Figure 4. As observed, in all cases, the extreme quasi-static response calculated from the effective node elevation and the effective water depth are between those from the Wheeler (lowest) and the vertical stretching methods (highest). This is because water particle kinematics from the Wheeler stretching method are lower than those that those from the vertical stretching method.

In view of the general belief that the vertical stretching method can overpredict the responses, the effective water depth procedure seems to be more suitable for practical application. Overall, it can be concluded that the difference between 100-year predictions from the Wheeler and the vertical stretching are too large to be neglected and therefore, further investigation is necessary to resolve this problem.

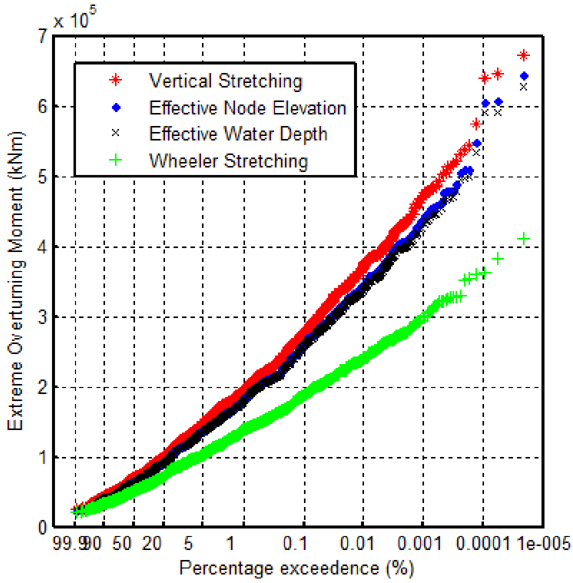

Figure 3: Comparison of probability distributions of extreme values of quasi-static overturning moment from 4 different methods of simulating water particle kinematics, 20000 sample records, T = 128sec. H_s = 5m, Tz = 7.94sec.

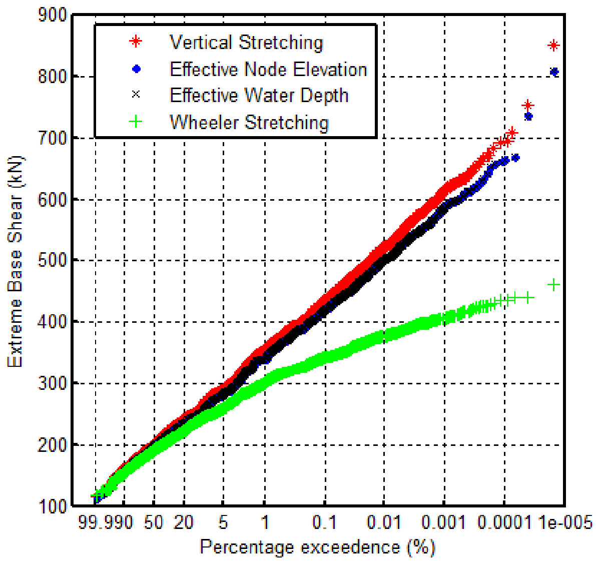

Figure 4: Comparison of probability distributions of extreme values of quasi-static base shear from 4 different methods of simulating water particle kinematics, 20000 sample records, T = 128sec. H_s = 5m, Tz = 7.94sec.

8 CONCLUSIONS

- Linear random wave theory (LRWT) is well known to lead to water particle kinematics with exaggerated high frequency components in the vicinity of MWL. To avoid this problem, modified versions of LRWT, such as Wheeler, vertical stretching, effective node elevation and effective water depth methods, are used to prevent this problem. Each of these methods is intended to calculate sensible kinematics above the MWL, yet they have been found to differ from one another in their predictions.

- Comparison between different methods make it clear that water particle kinematics from the effective water depth and effective node elevation methods in the near surface zone lie between those from the Wheeler and the vertical stretching methods. This is promising as there is some evidence that the water particle kinematics under crests are underestimated by the Wheeler method and that those from the vertical stretching method are somewhat exaggerated.
- The current investigation shows that the probability distributions of extreme responses based on the Wheeler and the vertical stretching methods can be significantly different from each other, leading to uncertainty as to which method should be used in design. Further research is therefore required to resolve this issue.
- It would be desirable to compare the results from these methods with high-quality laboratory and field data to observe how well they compare with measured data. Alternatively, they could be compared with water particle kinematics simulated from the more accurate Hybrid Wave Model.

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